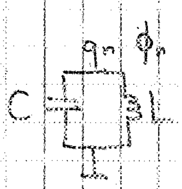
1st example: LC oscillator



$i_C = C \frac{d}{dt} V_C \qquad \frac{di_L}{dt} L = V_L$

Kirchhoff: $\quad i_C + i_L = 0 \qquad V_C = V_L$

$\phi_n = \int V\,dt = L\, i_L = L I$

$q_n = \int i\, dt = C U$

$$\mathcal{L}(q, \dot q) = \frac{q^2}{2C} - \frac{L}{2} \dot q^2$$

$$0 = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{d}{dt}\left(L \dot q\right) + \frac{q}{C} = -L\ddot q - \frac{1}{C} q$$

$$\mathcal{H}(q, \phi) = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

$$\hat{\mathcal{H}}(\hat q, \hat\phi) = \frac{1}{2}\frac{\hat q^2}{C} + \frac{1}{2}\frac{\hat\phi^2}{L} \quad \text{harmonic oscillator}$$

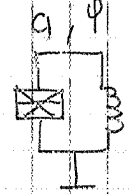
$$[\hat q, \hat \phi] = i\hbar$$

$$\hat q = \sqrt{\frac{\hbar}{2 Z_c}}\,(a^{+} + a) \qquad \hat a = \frac{1}{\sqrt{2\hbar Z_c}}\,(Z_c \hat q + i \hat \phi)$$

$$\hat \phi = i\sqrt{\frac{\hbar Z_c}{2}}\,(a^{+} - a) \qquad \hat a^{+} = \frac{1}{\sqrt{2\hbar Z_c}}\,(Z \hat q - i \hat \phi)$$

$$\hat{\mathcal{H}} = \hbar\omega\left(a^{+}a + \frac{1}{2}\right) \qquad Z_c = \sqrt{\frac{L}{C}}$$

RF-SQUID



$$\mathcal{H}(q, \varphi) = \mathcal{H}_{ind} + \mathcal{H}_{JJ}$$

$$\mathcal{H}_{ind} = \frac{\phi^2}{2L}$$

$$\mathcal{H}_{JJ} = \frac{q^2}{2C} - E_J \cos(\delta)$$

$$V_{JJ} = \frac{\phi_0}{2\pi} \dot{\delta}$$

$$\phi = \int_{ind} V_{ind} \, d\tau$$

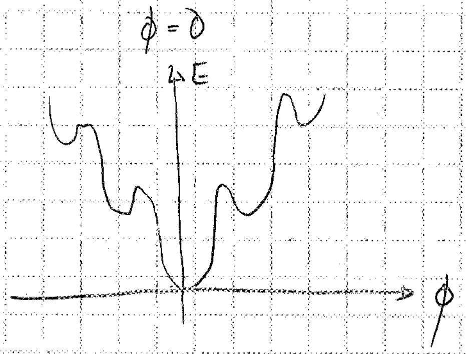
$$\phi_{ext} = \oint V = \phi - \underbrace{\int V_J \, d\tau}_{\frac{\phi_0}{2\pi}\delta}$$

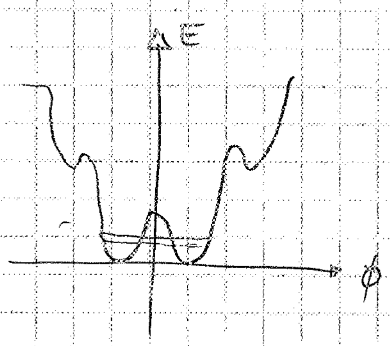
$$\mathcal{H} = \frac{q^2}{2C} + \frac{\phi^2}{2L} - E_J \cdot \cos\left(\frac{2\pi}{\phi_0}(\phi - \phi_{ext})\right)$$

problem has 2 parameters

E_J / E_C $\quad \frac{L_J}{L}$

no analytical solution

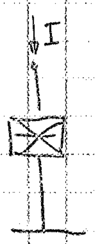




problem: qubit very sensitive to flux noise

$\boxed{\text{Slides}}$

Phase Qubit



current biased junction

$$\mathcal{H} = \frac{q^2}{2C} - I\phi - I\phi_0 \cos\left(\phi \frac{2\pi}{\phi_0}\right)$$

$$\delta = 2\pi \frac{\phi}{\phi_0} \qquad p = 2eq$$

$$\mathcal{H} = 4E_c\, p^2 - I\phi_0 \delta - I\phi_0 \cos(\delta)$$

$$[\delta, p] = i$$

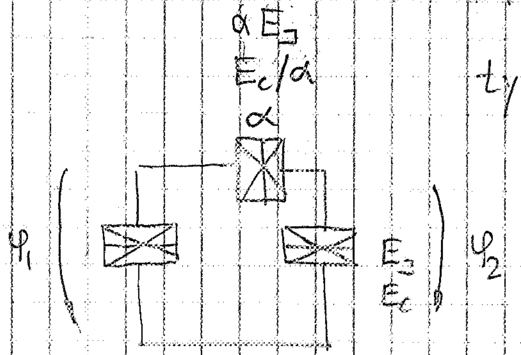
for $I \lessapprox I_0$

$$U(\delta) = \phi_0 (I_0 - I)(\delta - \pi/2) - \frac{I_0 \phi_0}{2\pi} (\delta - \pi/2)^3$$

$$\omega_p = \frac{1}{\sqrt{L_J(I) C}} = \frac{1}{\sqrt{L_J C_J}} \left(1 - (I/I_0)^2\right)^{1/4}$$

L10.1

persistent-current qubit

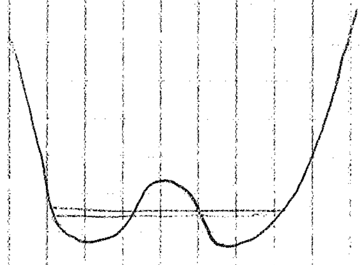


typical $\alpha = 0{,}8$

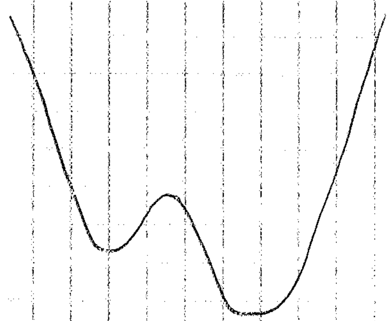
$$f = \frac{\Phi}{\Phi_0}$$

$$U = E_J \left[-\cos\varphi_1 - \cos\varphi_2 - \alpha \cos\left(2\pi f + \varphi_1 - \varphi_2\right)\right]$$

$f = 0{,}5$

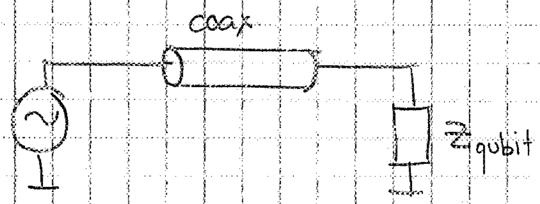


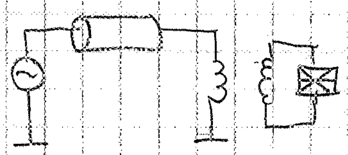
$f = 0{,}55$



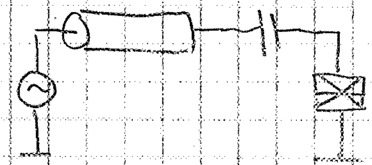
Qubit relaxation / dephasing

coupling to environment

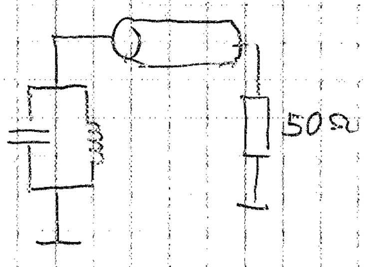


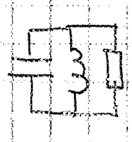


phase qubit



Cooper-pair box





RLC circuit

$$\omega = \frac{1}{\sqrt{LC}} \qquad \tau = \frac{1}{RC}$$

$$\propto e^{-\frac{t}{\tau}}$$

exponential decay

Fermi's golden rule

$$\Gamma = \frac{2\pi}{\hbar} \left|\langle f | H' | i \rangle\right|^2 \rho$$

L10.3

noise & dissipation

random signal $V(t)$

auto correlation

$$G_{VV}(t-t') = \langle V(t)\, V(t') \rangle = \langle V(t-t')\, V(0) \rangle$$

assumed stationary noise

$$\tilde{V}(\omega) = \int e^{i\omega t}\, V(t)\, dt$$

power spectral density

$$S_{VV}(\omega) = \langle |\tilde{V}(\omega)|^2 \rangle$$

since $V(t)$ is real
$S_{VV}(-\omega) = S_{VV}(+\omega)$

Wiener - Kinchin theorem

$$S_{VV} = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\, G_{VV}(t)$$

example: white noise

$$G_{VV}(t) = \sigma^2 \delta(t)$$

$$S_{VV}(\omega) = \sigma^2$$

quantum noise

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle \hat{x}(t)\, \hat{x}(0) \rangle$$

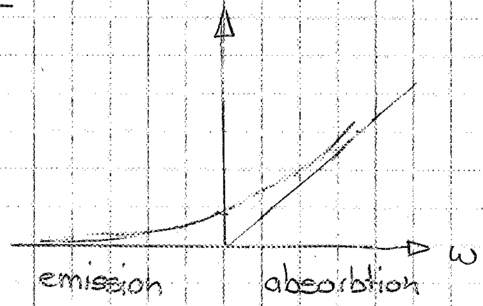
since $\hat{x}$ is an operator

$$S_{xx}(-\omega) \neq S_{xx}(+\omega)$$

L10.4

quantum noise of a resistor

$$S_{VV}(\omega) = \frac{2R\,\hbar\omega}{1 - e^{-\hbar\omega/k_BT}}$$



for $k_BT \gg \hbar\omega$ ('classical limit')

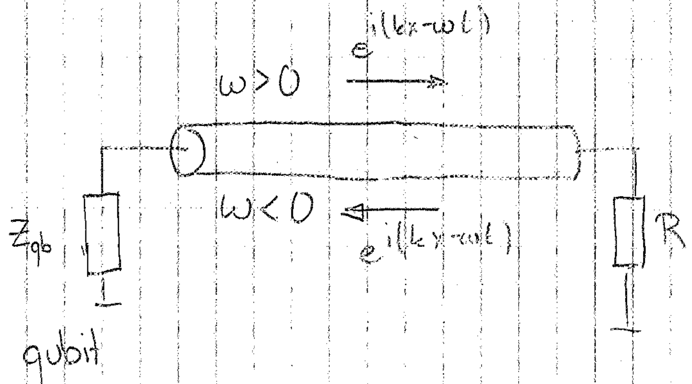
$$S_{VV}(\omega) = 2R\,k_BT$$

Johnson noise formula $S_{VV}(\omega) + S_{VV}(-\omega)$

'quantum limit'

Heavy-side function

$$S_{VV}(\omega) = 2R\,\hbar\omega\,\theta(\omega)$$



coupling to two-level system

$$H_0 = \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z$$

$$H' = A\,F(t)\,\hat{\sigma}_x$$

perturbation theory, first order (Fermi's golden rule)

$$\Gamma_\downarrow = \frac{A^2}{\hbar^2} S_{FF}(+\omega_{01})$$

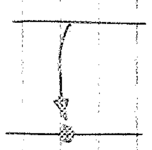
$$\Gamma_\uparrow = \frac{A^2}{\hbar} S_{FF}(-\omega_{01})$$

dephasing

$$\Gamma_\phi \propto S_{FF}(\omega \approx 0)$$

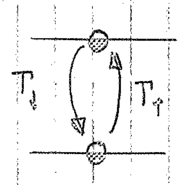
special form of the dissipation-fluctuation theorem

$k_B T \ll \hbar\omega_0$



qubit relaxes to ground state

$k_B T \gg \hbar\omega_0$



$\Gamma_\downarrow \approx \Gamma_\uparrow$

qubit is in an equal mixture of ground and excited state